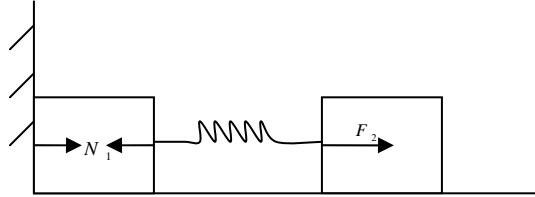


KUNCI OSN 2006 SEMARANG

1. a). Saat M_2 ditekan, pegas akan memberi gaya pada M_2 . Begitu juga M_2 akan memberi gaya reaksi pada pegas yang akan diteruskan ke M_1 dengan besar yang sama.



- $F_1 = F_2$
- Karena M_1 ditahan dinding, maka

$$N_1 = F_2$$

- Syarat M_1 lepas dari dinding

$$N_1 = 0$$

Sehingga

$$N_1 = F_2 = k \cdot \Delta x = 0$$

$\Delta x = 0$ (pegas kendur pertama kali)

Balok M_1 lepas dari dinding setelah $\frac{1}{4} T$

$$\begin{aligned} t_1 &= \frac{1}{4} T \\ &= \frac{1}{4} \left(2\pi \sqrt{\frac{m}{k}} \right) \\ t_1 &= \frac{\pi}{2} \sqrt{\frac{m}{k}} \end{aligned}$$

- b). Proses selanjutnya osilasi 2 massa

- Kerangka acuan bumi :
 - m_2 bergerak ke kanan dengan kecepatan v_{20}
 - m_1 diam.
 - kecepatan pusat massa

$$v_{cm} = \frac{mv_{20}}{m+m} = \frac{1}{2} v_{20}$$

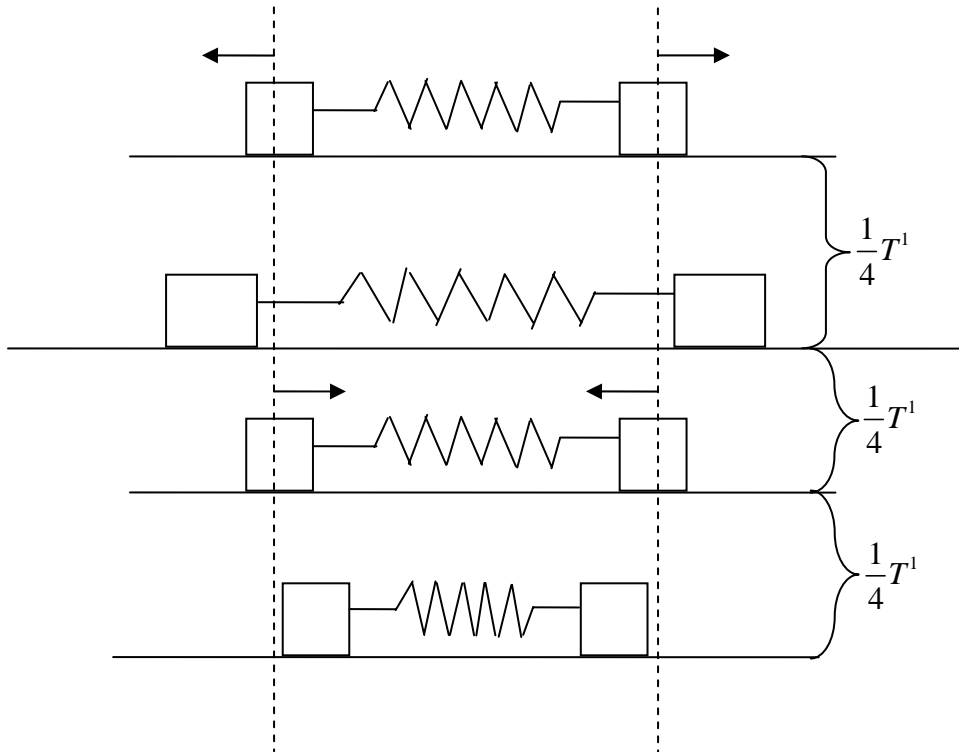
- Dalam kerangka pusat massa

- m_2 bergerak ke kanan dengan laju $\frac{1}{2} v_{20}$
- m_1 bergerak ke kiri dengan laju $\frac{1}{2} v_{20}$

- pusat massa diam

⇒ Periode osilasi = periode osilasi satu benda m dengan pegas $k' = 2k$

$$T' = 2\pi\sqrt{\frac{m}{2k}}$$



$$t_2 = t_1 + \frac{3}{4}T'$$

$$= \frac{\pi}{2}\sqrt{\frac{m}{k}} + \frac{3}{4}2\pi\sqrt{\frac{m}{2k}}$$

$$t_2 = \frac{\pi}{2}\sqrt{\frac{m}{k}} + \frac{3\pi}{2}\sqrt{\frac{m}{2k}}$$

c) Saat $t = t_1$

$$\frac{1}{2}kA^2 = \frac{1}{2}mv_{20}^2$$

$$v_{20} = A\sqrt{\frac{k}{m}}$$

Jarak terdekat

$$v_x = \frac{1}{2}v_{20}$$

$$v_x = \frac{A}{2}\sqrt{\frac{k}{m}}$$

kekekalan energi

$$2 \frac{1}{2} m v_x^2 = \frac{1}{2} k \Delta \ell^2$$

$$2 m \frac{A^2}{4^2} = \frac{k}{m} k \Delta \ell^2$$

$$\Delta \ell = \frac{A}{2} \sqrt{2}$$

$$\begin{aligned} \text{Jarak dekat} &= L - \Delta \ell \\ &= L - \frac{A}{2} \sqrt{2} \end{aligned}$$

d) Setelah t_1 , pusat massa bergerak menjauhi dinding dengan laju tetap $\frac{1}{2} v_{20}$

- Saat $t = t_1$

$$\text{Jarak pusat massa ke dinding} = \frac{L}{2}$$

- Saat $t = t_2$

Jarak pusat massa ke dinding .

$$\begin{aligned} x_2 &= \frac{L}{2} + \frac{1}{2} v_{20} (t_2 - t_1) \\ &= \frac{L}{2} + \frac{1}{2} A \sqrt{\frac{k}{m}} \left(\frac{3\pi}{2} \sqrt{\frac{m}{2k}} \right) \end{aligned}$$

$$x_2 = \frac{L}{2} + \frac{3\pi}{8} A \sqrt{2}$$

Jarak benda pertama ke pusat massa

$$\begin{aligned} x_1 &= \frac{1}{2} \times \text{jarak terdekat} \\ &= \frac{L}{2} - \frac{A}{4} \sqrt{2} \end{aligned}$$

Jarak benda pertama ke dinding

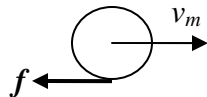
$$\begin{aligned} \Delta x &= x_2 - x_1 \\ &= \frac{3\pi}{8} A \sqrt{2} + \frac{A}{4} \sqrt{2} \\ &= \frac{A \sqrt{2}}{8} (3\pi + 2) \end{aligned}$$

02.

Tinjau bola sebelum menggelinding.



Tinjau bola setelah menggelinding (masuk daerah kasar)



Gaya gesek f memperlambat bola dan membuat bola berotasi.

- Perlambatan a_m :

$$a_m = -\frac{f}{m}$$

- Kecepatan bola v_m :

$$v_m = v_0 + at$$

$$v_m = v_0 - \frac{f}{m}t \dots\dots\dots (1)$$

- Rotasi bola :

$$\Sigma \tau = I\alpha$$

$$f r = I\alpha$$

$$f r = I \frac{\omega_m}{t}$$

$$\omega_m = \frac{f r}{I}t \dots\dots\dots (2)$$

Tinjau kereta (massa M).

Gaya gesek mempercepat kereta (a_M) :

- Percepatan kereta a_M :

$$a_M = \frac{f}{M}$$

- Kecepatan kereta (v_M) :

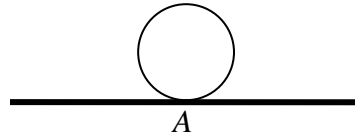
$$v_M = v_0 + at$$

$$v_M = 0 + \frac{f}{M}t$$

$$v_M = \frac{f}{M}t \quad \dots\dots\dots (3)$$

Syarat bola menggelinding tanpa slip :

- Kecepatan titik A = kecepatan kereta ($v_A = 0$)



Kecepatan titik A = kecepatan titik A terhadap pusat bola + kecepatan pusat bola terhadap tanah.

$$v_M = v_m - \omega_m r \quad \dots\dots\dots(4)$$

Dari empat persamaan di atas, didapatkan :

$$\frac{f}{M}t = (v_0 - \frac{f}{m}t) - \frac{f}{I}t r$$

$$t = \frac{2mM}{2m + 7M} \frac{v_0}{f}$$

- Kecepatan kereta v_M :

$$v_M = \frac{2m}{2m + 7M} \frac{v_0}{r}$$

- Kecepatan linear bola v_m :

$$v_m = \frac{2m + 5M}{2m + 7M} v_0$$

- Kecepatan angular bola ω_m :

$$\omega_m = \frac{5M}{2m + 7M} \frac{v_0}{r}$$

b). Jarak minimum s (jarak relatif bola terhadap kereta) :

$$s = (v_0 t - \frac{1}{2} \frac{f}{m} t^2) - (\frac{1}{2} \frac{f}{M} t^2)$$

$$= \frac{2M(m + 6M)}{(2m + 7M)^2} \frac{v_0^2}{\mu g}$$

3. a). Saat M bergeser ke kanan sejauh x ,
 Posisi m berubah sebagai berikut :
 Arah horizontal : $x - x \cos \alpha$
 Arah vertikal : $-x \sin \alpha$

Sehingga :

$$a_{mx} = a_M (1 - \cos \alpha) \dots\dots\dots 1$$

$$a_{my} = -a_M \sin \alpha \dots\dots\dots 2$$

➤ Tinjau M

Arah horizontal

$$T - T \cos \alpha + N \sin \alpha = M a_M \dots\dots\dots 3$$

➤ Tinjau m

Arah horizontal :

$$T \cos \alpha - N \sin \alpha = m a_{mx} \dots\dots\dots 4$$

Arah vertikal :

$$T \sin \alpha - mg + N \cos \alpha = m a_{my} \dots\dots\dots 5$$

➤ Substitusi 1 → 4

$$T \cos \alpha - N \sin \alpha = m a_M (1 - \cos \alpha) \dots\dots\dots 6$$

➤ Substitusi 2 → 5

$$T \sin \alpha - mg + N \cos \alpha = -m a_M \sin \alpha \dots\dots\dots 7$$

➤ Persamaan 3 + 6

$$T = M a_M + m a_M (1 - \cos \alpha) \dots\dots\dots 8$$

➤ (persamaan 6 dikalikan $\cos \alpha$) + (persamaan 7 dikalikan $\sin \alpha$)

$$T - mg \sin \alpha = m a_m (1 - \cos \alpha) \cos \alpha - m a_M \sin^2 \alpha \dots\dots\dots 9$$

➤ dari persamaan 8 dan 9

$$M a_M + 2 m a_M (1 - \cos \alpha) = m g \sin \alpha$$

$$a_M = \frac{m g \sin \alpha}{M + 2m (1 - \cos \alpha)} \dots\dots\dots 10$$

b). Substitusi 10 → 8

$$T = \frac{M + m(1 - \cos \alpha)}{M + 2m (1 - \cos \alpha)} m g \sin \alpha$$

➤ energi kekal karena tegangan tali tidak melakukan usaha (constraint force).

c.) Hukum kekal energi (Saat M bergeser x_o)

$$EP_m = - m g x_o \sin \alpha$$

$$EP_{\text{pgs}} = \frac{1}{2} k x_o^2$$

$$-mgx_o \sin \alpha + \frac{1}{2} k x_o^2 = 0$$

$$x_o = \frac{2mg}{k} \sin \alpha$$

d). Pada posisi setimbang $\Sigma F = 0$

Tinjau M :

Arah horizontal

$$T - T \cos \alpha + N \sin \alpha - k\Delta x = 0 \dots\dots\dots a$$

Tinjau m :

Arah horizontal

$$T \cos \alpha - N \sin \alpha = 0 \dots\dots\dots b$$

Arah vertikal

$$T \sin \alpha - mg + N \cos \alpha = 0 \dots\dots\dots c$$

Dari persamaan b dan c didapat :

$$T = mg \sin \alpha$$

$$N = mg \cos \alpha \dots\dots\dots d$$

➤ Substitusi d → a

$$\Delta x = \frac{mg}{k} \sin \alpha$$

4. Percepatan setelah mantul

$$a_x = g \sin \theta$$

$$a_y = -g \cos \theta$$

Kecepatan

$$v_x = v_o \sin \theta$$

$$v_y = v_o \cos \theta$$

Waktu untuk satu kali tumbukan :

$$t = \frac{2v_y}{-a_y} = \frac{2v_o \cos \theta}{g \cos \theta} = \frac{2v_o}{g}$$

Waktu 2 kali tumbukan

$$t = \frac{4v_o}{g}$$

Jarak mendatar

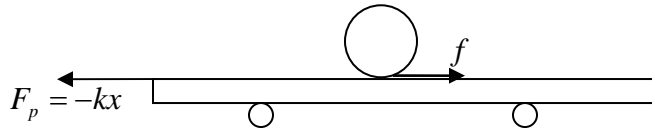
$$\square x = v_x t + \frac{1}{2} a_x t^2$$

$$d_{12} = v_o \sin \theta \left(\frac{2v_o}{g} \right) + \frac{1}{2} g \sin \theta \left(\frac{4v_o^2}{g^2} \right) = \frac{4v_o^2 \sin \theta}{g}$$

$$d_{13} = v_o \sin \theta \left(\frac{4v_o}{g} \right) + \frac{1}{2} g \sin \theta \left(\frac{16v_o^2}{g^2} \right) = \frac{12v_o^2 \sin \theta}{g}$$

$$\frac{d_{12}}{d_{23}} = \frac{1}{2}$$

5. a.) Tinjau saat kereta dipercepat ke kiri



➤ Persamaan gerak kereta

$$\sum F = M a_M$$

$$f - kx = M a_M \quad \dots\dots\dots 1$$

➤ Percepatan bola terhadap tanah :

$$a_m = a_{mM} + a_M \quad \dots\dots\dots 2$$

➤ Percepatan sudut bola

$$\alpha_m = \frac{a_{mM}}{r} \quad \dots\dots\dots 3$$

➤ Persamaan gerak bola

$$\sum F = m a_m$$

$$-f = m(a_{mM} + a_M) \quad \dots\dots\dots 4$$

$$\sum \tau = I \alpha_m$$

$$f r = I \alpha_m \quad \dots\dots\dots 5$$

➤ 3→5

$$f r = \frac{2}{5} m r^2 \frac{a_{mM}}{r}$$

$$f = \frac{2}{5} m a_{mM} \quad \dots\dots\dots 6$$

➤ substitusi 6→4

$$a_M = -\frac{7}{5} a_{mM} \quad \dots\dots\dots 7$$

➤ sehingga

$$f = -\frac{2}{7} m a_M \quad \dots\dots\dots 8$$

➤ substitusi 8→1

$$0 = \frac{2}{7} m a_M + M a_M + kx$$

$$0 = \left(\frac{2}{7} m + M\right) a_M + kx$$

$$0 = \left(\frac{2m+7M}{7k}\right)a_M + x$$

$$0 = a_M + \left(\frac{7k}{2m+7M}\right)x \quad \dots\dots\dots 9$$

$$\omega^2 = \frac{7k}{2m+7M}$$

$$T = 2\pi\sqrt{\frac{2m+7M}{7k}} \quad \dots\dots\dots 10$$

b). Syarat tergelincir

$$f = \mu m g \quad \dots\dots\dots 11$$

karena $f = -\frac{2}{7}ma_M$, maka

$$a_m = -\frac{7}{2}\mu g \text{ (percepatan maksimum tanpa slip)}$$

dari persamaan 9

$$\frac{7}{2}\mu g = \frac{7k}{2m+7M} x_{\max}$$

$$x_{\max} = \frac{2m+7M}{2} \frac{\mu g}{k} \quad \dots\dots\dots 12$$